MATH-SHU 350

Recitation 11: Weak Convergence

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Exercise 1. *Prove that random variable X is symmetric (X and $-X$ have the same law) if and only if its characteristic function φ_X takes real value.*

Exercise 2. *Let $X \sim \mathcal{N}(0, \sigma^2)$ and Φ its characteristic function.*

1. *Prove that $\Phi'(t) = -t\sigma^2\Phi(t)$;*
2. *Calculate $\Phi(t)$.*

Exercise 3. *Calculate the characteristic function for the random variable X if*

1. *X follows Bernoulli distribution of parameter $p \in (0, 1)$;*
2. *X follows Binominal distribution of parameter (n, p);*
3. *X follows Poisson distribution of parameter λ;*
4. *X follows exponential distribution of parameter θ;*
5. *X follows symmetric exponential distribution of density $f(y) = \frac{\lambda}{2}e^{-\lambda|y|}$;*
6. *X follows Cauchy distribution of density $f(x) = \frac{\alpha}{\pi(\alpha^2+x^2)}$.*

Exercise 4. *Prove that if $(X_n)_{n\in\mathbb{N}}$ satisfies uniform integrability, then they are tight.*

Exercise 5 (Slutsky's theorem)**.** *If X_n converges in distribution to X and Y_n converges in distribution to a constant c, then the joint vector (X_n, Y_n) converges in distribution to (X, c).*